Algorithm for Queue Estimation with Loop Detector of Time Occupancy in Off-Ramps on Signalized Motorways

Gongbin Qian, Jinwoo (Brian) Lee, and Edward Chung

Long traffic queues on off-ramps significantly compromise the safety and throughput of motorways. Obtaining accurate queue information is crucial for countermeasure strategies. However, it is challenging to estimate traffic queues with locally installed inductive loop detectors. This paper deals with the problem of queue estimation with the interpretation of queuing dynamics and the corresponding time-occupancy distribution over motorway off-ramps. A novel algorithm for real-time queue estimation with two detectors is presented and discussed. Results derived from microscopic traffic simulation validated the effectiveness of the algorithm and revealed some of its useful features: (*a*) long and intermediate traffic queues could be accurately measured, (*b*) relatively simple detector input (i.e., time occupancy) was required, and (*c*) the estimation philosophy was independent with signal timing changes and provided the potential to cooperate with advanced strategies for signal control. Some issues concerning field implementation are also discussed.

Growing traffic demand on motorways has increased the occurrence of congestion on off-ramps. The congestion on off-ramps, forming long queues and queue spillover onto the motorway main line, significantly compromises the safety and throughput of the motorway. To develop any queue management strategy, it is necessary to obtain the capability to accurately estimate the queue information on off-ramps.

Although the use of video sensors could be one approach for capturing queue information, their application may be affected by various factors such as weather conditions, visibility obstacles, and the reliability of imaging processing techniques. In practice, inductive loop detectors are much more widely used to measure queue information. A practical approach usually uses one or more loop detectors at the upstream of a link as a queue regulator. As inductive loop detectors are able to provide flow, speed, and occupancy measurements, usually a certain occupancy or speed value is predefined to represent that a queue has reached the location of the detector. For instance, observing 100% occupancy measurements indicates a stopped vehicle over the detector. This method, however, cannot clearly capture the queue dynamics (i.e., queue length over time) before and after the queue reaches the detecting area, which is crucial information for more advanced queue management strategies.

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Aiming at a better presentation of off-ramp queue dynamics based on loop detector measurements, this paper presents a real-time queue estimation algorithm. The primary objectives of the algorithm are to (*a*) accurately describe real-time queue dynamics especially for long and intermediate vehicle queues, (*b*) efficiently deploy loop locations and make good use of detector information, and (*c*) make the algorithm independent from traffic signal timing changes at a downstream intersection.

This paper is organized as follows. First, the existing queue estimation techniques for signalized traffic links are reviewed. Then a discussion is presented on the relationship between queue dynamics and time occupancies measured by loop detectors, which will provide the theoretical framework of the proposed queue estimation method and algorithm. The underlying principles and details of the algorithm are presented next, followed by a discussion on the simulation-based evaluation results of the proposed algorithm. Conclusions and some issues for field implementation are presented last.

TECHNIQUES OF QUEUE ESTIMATION

Abundant literature about queue estimating techniques exists, as it is always challenging to obtain accurate queue information with limited data. The conventional induced loop detectors are able to provide three primary kinds of measurement, namely vehicle count, vehicle speed, and occupancy of detector. As a result, the existing queue estimation techniques deploy various loop settings and detector measurements.

Webster developed a classic method using the input-and-output accumulation counts of vehicle count to estimate the number of vehicles in a signalized link (*1*). A similar approach was adopted by Sharma et al. (*2*). Both of the studies reported reasonably good results. However, it was found that with increased link lengths, the estimation accuracy declined substantially. In addition, from a practical point of view, the input-and-output approach is subject to the detector's counting errors, which accumulate into large errors over a long analytical period without continuous calibration.

Using occupancy information is another approach. Papageorgiou and Vigos first explored the relationship between time occupancies and space occupancies in signalized links (*3*). The research elaborately interpreted local occupancy measurement into number of queuing vehicles over the whole link during each detection interval, which avoided accumulative computations. The study proposed that the space occupancy measured by a middle-block loop detector can be an approximation for the space occupancy in the entire link. Tests in simulation indicated that the estimation accuracy is subject to a variety of factors, including the number of detectors, estimation intervals, and traffic signal timings. This method is highly sensitive to the signal control at the downstream intersection. This may impede the application of the method for traffic-response control strategies. Vigos et al. (*4*) and Mucsi et al. (*5*) followed a similar approach to analyze queue dynamics and reported that the model reliability is heavily affected by variable traffic signal timings.

Vigos and Papageorgiou integrated the occupancy-based queue estimation method with the input-and-output method by using the Kalman filtering signal processing technique (*6*). The study attempted to calibrate the accuracy of the input-and-output method by using the occupancy date measured by an extra loop detector placed in the middle of the link. This study suggested that queue estimation could be more accurate with additional loop detectors placed between the link entrance and the stop line.

A more comprehensive analysis and integration of loop detector information was shown in the study by Liu et al. (*7*). The kinematic features of traffic flows (i.e., queuing behaviors) before stop lines are projected on the basis of the information collected from a detector located in the middle of the link. Although this method demonstrated reliable estimation results, it requires high-resolution data, such as second-by-second occupancy, vehicle speed, and traffic flow measurements, which are normally difficult to obtain in prevailing traffic control systems. In addition, this method cannot handle long queues in oversaturation conditions, and the detector must be relocated to a further upstream position in long queue scenarios.

Overall, the accuracy of the existing queue estimation techniques is highly dependent on the quantity of traffic detectors, the quality of data measurements, and the detector locations. The traffic characteristics on motorway off-ramps are different from typical arterials where the upstream traffic signals mainly induce the vehicle arrival patterns. The following sections will further investigate an appropriate queue estimation technique for motorway off-ramps.

ANALYSIS OF TIME OCCUPANCY

Spatial-dimension measurement, such as density and space occupancy, is important information for queue estimation. However, density and space occupancy are not directly available from conventional inducted loop detectors; instead, temporal-dimension measures such as time occupancy are usually available. This section will explore the local relationship between time occupancy and space occupancy in signalized links.

Time Occupancy and Local Density

The time occupancy, O_t, is a direct measurement obtained from loop detectors. Time occupancies are computed as the proportion of the time period in which the detector is occupied by vehicles during a time horizon, Δt.

$$O_t = \sum_{i=1}^{N} \frac{t_i}{\Delta t} = \sum_{i=1}^{N} \frac{\frac{L_i}{v_i}}{\Delta t} \tag{1}$$

where

- N = total number of vehicles that passed detector during Δt,
- t_i = time that the loop is occupied by ith vehicle,
- L_i = effective vehicle length of i, and
- v_i = corresponding vehicle speed of i.

The variable t_i can be computed by dividing the effective vehicle length of L_i by the corresponding vehicle speed, v_i. The effective vehicle length does not equal the physical vehicle length, L_i^{ph}, because a detector is activated after the head of a vehicle enters the detecting area until the rear of a vehicle leaves. As a result,

$$L_i = L_i^{ph} + L_d \tag{2}$$

where L_d is the length of the loop detector.

Now the time occupancy, O_t, collected from a single loop detector can be described as

$$O_t = \sum_{i=1}^{N} \frac{L_i^{ph} + L_d}{v_i \Delta t} \tag{3}$$

All variables in Equation 3 are described in relation to spatial parameters. According to Wardrop, with the assumption of homogenous traffic flow over the whole link, the local density measure, ρ, within a sufficiently small substream can be expressed as $\rho = 1/v\Delta t$, where v is the speed of the substream vehicle and Δt is the analytical period (*8*). Applying Wardrop's approach, the bridge can be built between time occupancy, O_t, and link density, D, for a motorway lane, and the relation is shown as follows (*9, 10*):

$$O_t = \bar{L}D \tag{4}$$

$$\bar{L} = \frac{\sum_i \left(\frac{L_i}{v_i}\right)}{\sum_i \left(\frac{1}{v_i}\right)} \tag{5}$$

where $\bar{L}$ is the mean effective vehicle length and D is the link density.

Equations 4 and 5 indicate that the time occupancy is proportional to the traffic density if the effective vehicle length or the average vehicle speed are constant (*6*). If Equation 4 is applied on a relatively short roadway segment, then the time occupancy, O_t, during a short time and the instantaneous local density, ρ, are linear-related.

Time Occupancy in Signalized Links

In signalized links, traffic characteristics over segments are typically heterogeneous because of the traffic signal control at the upstream and downstream intersections. As a result, it is difficult to precisely obtain global spatial-dimension measures by using a limited number of local detectors. Consider a signalized link with length R. Divide the link by small segments with each segment length of d as shown in Figure 1.

The total number of segments is R/d. Assuming hypothesized detectors in all the small segments in the link, suppose O_t^m denotes the measured time occupancy from the segment m during time Δt. Spatially aggregating O_t^m over the link, a new measurement, $\bar{O}$, can be generated. The local time occupancy, O_t^m, can be replaced with $\bar{L}D$ according to Equation 4 and then $\bar{O}$ can be computed:

$$\bar{O} = \sum_{m=1}^{\frac{R}{d}} O_t^m \approx \sum_{m=1}^{\frac{R}{d}} \bar{L}\rho^m = \alpha D \tag{6}$$

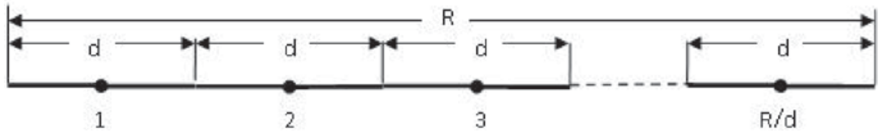

FIGURE 1 Segments within traffic link.

where

O_t^m = time occupancy of m segment during Δt;
m = segment index, $m = 1, 2, \ldots, R/L_d$; and
α = linear coefficient.

Equation 6 converts the local time occupancy, O_t^m to the link density, D. However, it is an implausible assumption that the individual segment occupancy O_t^m is available. The next section introduces a practical approach to estimate $\bar{O}$ and the link density D.

DEVELOPMENT OF ALGORITHM FOR QUEUE ESTIMATION

This section presents a method to estimate $\bar{O}$ by using the time occupancy measurements from two loop detectors: one is in the middle of the link and the other is at the link entrance. The positions of two loop detectors were determined to ensure the reliable queue estimation for long and intermediate queues, which is of crucial importance for traffic management strategies to prevent queue spillover. Having more detectors will improve the estimation accuracy but at increased cost. A single detector setting will provide reliable estimations for only a short segment.

QUEUING FEATURES ON SIGNALIZED OFF-RAMPS

On signalized off-ramps, different vehicle behaviors may be observed because the queue size is highly dependent on the traffic signal control at the downstream intersection. During the red signal phase, vehicles will start forming a queue from the stop line and three different vehicle behaviors based on the position of the vehicles against the traffic queue can be observed. Vehicles in the queue will creep or completely stop. Vehicles approaching the queue will have to significantly reduce their speed. Vehicles in a free-flow condition will experience neither deceleration nor stopping if the downstream traffic condition is clear, say, the traffic queue or traffic signal is still far away.

If loop detectors were implemented in every short segment, the distribution of O_t^m over the whole link will be available. The subscript t can be several seconds, minutes, or hours, depending on the analytical period. The observed O_t^m may reflect the three vehicle behaviors mentioned previously. The detectors positioned far upstream will observe a relatively lower O_t^m as the vehicle near the detector may not be affected by either traffic signals or preceding vehicles, and thus a free-flow occupancy, $F[O_t^m]$, will be observed. Normally the driving speeds in the free-flow condition do not vary significantly among the vehicles and, thus, the value of $F[O_t^m]$ is relatively stable according to Equation 3.

Similarly, relatively higher O_t^m values will be observed from the hypothesized detectors near the stop line where the queue forms. Let $S[O_t^m]$ denote the queuing occupancy representing the occupancy measurements affected by stopping or slowly moving vehicles. Those downstream detectors under the stationary or slowly moving traffic queue will report $O_t^m \approx S[O_t^m]$.

In between $F[O_t^m]$ and $S[O_t^m]$, the assumed detectors will observe varied values of O_t^m because the vehicles passing those detectors will decelerate from a free-flow speed until encountering the end of the queue. The detectors on those segments in which decelerating vehicles are encountering the queue will observe relatively high values of O_t^m, close to $S[O_t^m]$ as the vehicles passing the segments will just start decelerating. Consequently, the occupancies observed between the free-flow regime and the queue regime will gradually decrease from $S[O_t^m]$ to $F[O_t^m]$, denoted by $U[O_t^m]$. As such, the distribution of time occupancy O_t^m along the link can be approximated on the basis of the aforementioned vehicle behaviors in three regimes as illustrated in Figure 2.

In Figure 2, Point A and Point B represent the position where regimes of vehicle behaviors change. Point A is the break point between $S[O_t^m]$ and $U[O_t^m]$, indicating the end of the stopping queue. Point B is the break point between $U[O_t^m]$ and $F[O_t^m]$, indicating the location where the approaching vehicles are affected by the queue and thus start to decelerate.

If the occupancy distribution over the link could be fully obtained, as shown in Figure 3, the area covered by the observed occupancy values (i.e., the shaded area) represents the spatially aggregated time occupancy, $\bar{O}$, which was introduced in the previous section.

As shown in Figure 3, the position of Point A and Point B and the value of $S[O_t^m]$ and $F[O_t^m]$ determine $\bar{O}$. The value of $\bar{O}$ can be computed by adding up the shaded area in Figure 3, as follows:

$$\bar{O} = F\left[O_t^m\right] \times L_b + \frac{1}{2}\left(F\left[O_t^m\right] + S\left[O_t^m\right]\right) \times L_{ab} + S\left[O_t^m\right] \times L_a \qquad (7)$$

where

L_a = distance between stop line and Point A,
L_{ab} = distance between Point A and Point B, and
L_b = distance between off-ramp entrance and Point B.

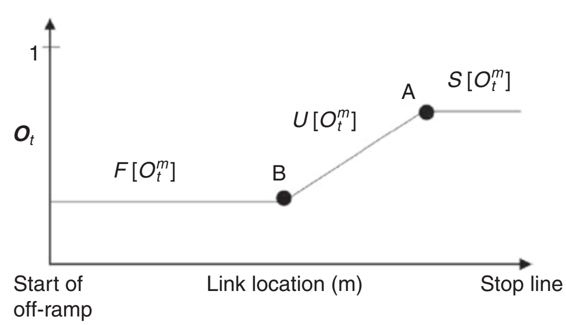

FIGURE 2 Distribution of O_t^m along off-ramp link.

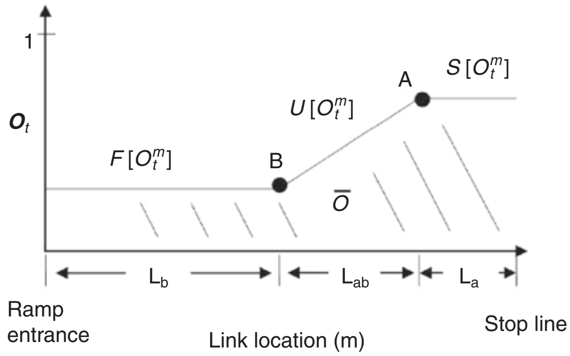

FIGURE 3 Distribution of O_t^m and $\bar{O}$.

Equation 7 is not a practical solution, because the exact positions of Point A and Point B are unknown and change over time. The proposed queue estimation algorithm approximates the occupancy distribution over the whole link by using the local occupancy measurements from two loop detectors. Figure 4 shows some examples of the queue estimation for different queue sizes.

In the figures, the midlink loop detector is denoted as H_1 and the other detector at the link entrance is denoted as H_2. Those H_1 and H_2 detectors report the time occupancy, O_1 and O_2, respectively. The solid lines indicate the actual occupancy distribution of O_t^m, and the dashed lines indicate the estimated occupancy distribution by the proposed algorithm.

In Figure 4*a,* a short queue formed at the stop line does not affect the detector measurements, O_1 and O_2. As the queue grows and approaches H_1, the vehicles passing H_1 need to reduce their speed, which results in the increased occupancy measurements, O_1, as displayed in Figure 4*b*. In Figure 4*c,* the queue end stands between H_1 and H_2. Since H_1 is under the queue, O_1 includes the queue occupancy, $S[O_t^m]$, while the traffic passing H_2 is yet unrestricted (i.e., $O_2 = F[O_t^m]$). Figure 4*d* shows a nearly saturated condition so that the queue impedes the upstream traffic. As a result, Point B has moved out of the link, implying that all vehicles in the link are affected by the queue. The upstream detector occupancy also approaches the queuing occupancy (i.e., $O_2 = F[O_t^m]$).

The proposed queue estimation method approximates the occupancy distribution by plotting a connection line between O_1 and O_2, and another horizontal line from O_1 to the stop line direction, displayed as the dashed lines in the figures. This processing assumes a uniform occupancy of O_1 for the downstream segments between H_1 and H_2, while $O_t^m = O_1 = O_2$ if $O_1 = O_2$. The area covered by the approximated occupancy distribution represents the estimated value of space aggregated occupancy, $\bar{O}'$.

Let a denote the link length of the upstream section between H_1 and H_2 and b denote the downstream section length between H_1 and the stop line. The area covered by the dashed lines, or $\bar{O}'$, can be computed with the following equation:

$$\bar{O}' \approx \frac{(a+2b)\times O_1 + a\times O_2}{2(a+b)} \tag{8}$$

With Equation 6 applied to Equation 8, the link density can be calculated as

$$D \approx \frac{(a+2b)\times O_1 + a\times O_2}{2\alpha(a+b)} \tag{9}$$

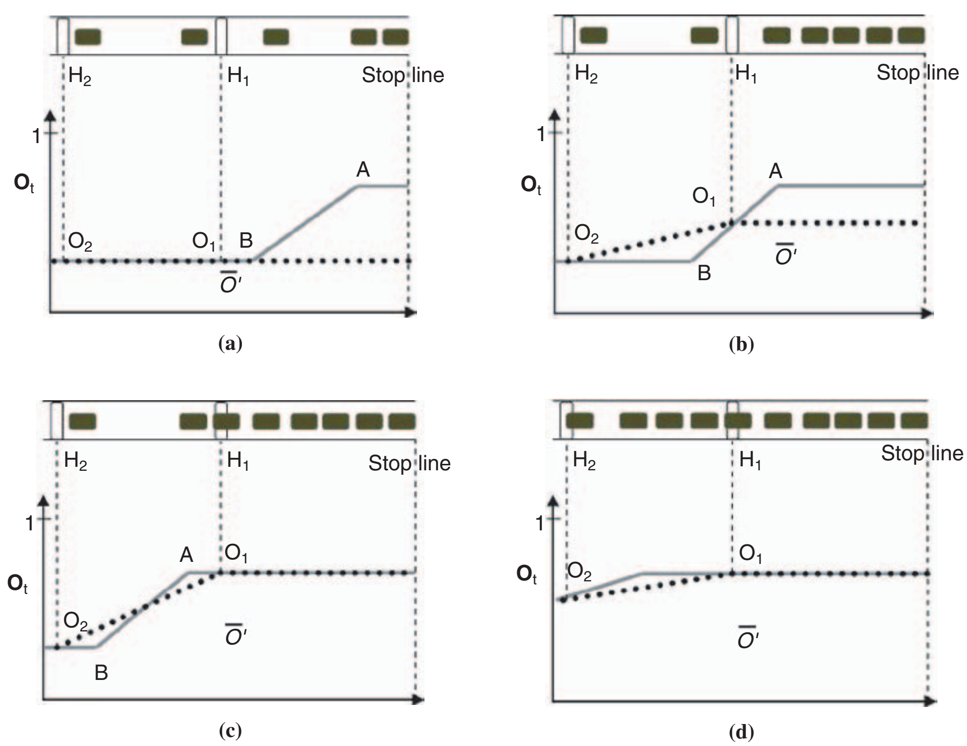

FIGURE 4 Queuing vehicles and occupancy approximation: (*a*) short queue at stop line, (*b*) growing queue at stop line with increased O_1 measurements, (*c*) $O_1 = S[O_t^m]$ and $O_2 = F[O_t^m]$, and (*d*) nearly saturated condition.

Eventually the link density is expressed by using two measurable quantities, namely, the local time occupancy measurements from H_1 and H_2 in Equation 9.

SIMULATION TEST

Simulation Design and Basic Results

The proposed estimation algorithm is tested with a microscopic traffic simulator, AIMSUN. A tested model was created by using the actual data collected from the Moggill Road off-ramp, located in Brisbane, Queensland, Australia. The collected data include the traffic counts at the entrance of the off-ramp and traffic signal timings at the downstream intersection. Figure 5 shows the simulation model of the Moggill Road interchange.

The selected off-ramp consists of three lanes, including one exclusive left-turning lane and two right-turning lanes. Each lane is approximately 100 m (328 ft) long. The off-ramp intersection operates a semi-actuated control; the green phase for the off-ramp is extended by the vehicle actuations. The maximum cycle time is relatively long at 140 s during the afternoon peak hours. To implement the queue estimation algorithm in the tested model, a midlink loop detector was installed 50 m upstream from the stop line and another detector was placed at the ramp entrance.

Before the queue estimation algorithm was tested, the linear coefficient α was predefined according to Equation 8. According to Equation 6, α can be computed by using the jam link density and the aggregated time occupancy ($\bar{O}$) corresponding to the jam density. Both the jam density and the corresponding time occupancy were derived from the simulation test results. These two parameters can be easily estimated from field measurements.

The simulation with the queue estimation algorithm was then executed for 1.5 h for the afternoon peak from 5:00 to 6:30 p.m. A total of 10 replications were run with random seeds. Figure 6 shows a comparison of the actual and the estimated number of vehicles in the link from the replication results, as an example. The estimation update interval is 15 s in the simulation results. In Figure 6, the queue dynamics are well described by the proposed algorithm over the entire simulation period. For short queues, the algorithm tends to underestimate the queue size on some occasions. The reason is that the current detector positioning was designed to capture long and intermediate traffic queues.

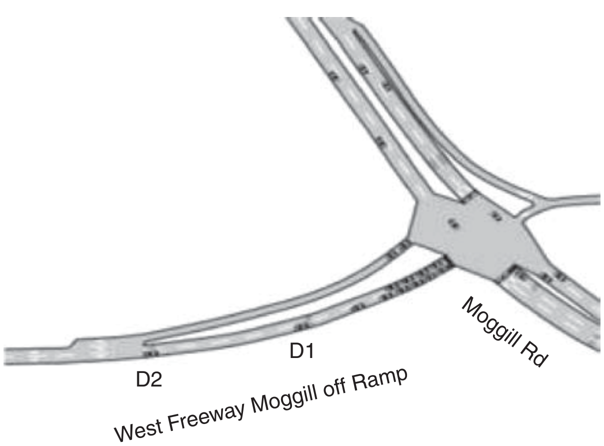

FIGURE 5 Test bed simulation network.

Sensitivity About Queue Size

The different performances of the algorithm queue size were demonstrated in Figure 6; a further investigation was conducted to analyze the capability for different queue sizes. Two performance measures are introduced: mean absolute percentage error (MAPE) and root mean square error (RMSE), with the following definitions:

$$\text{MAPE} = \frac{1}{n}\sum_{n}\left|\frac{(\text{observation} - \text{estimation})}{\text{observation}}\right| \times 100\% \tag{10}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{n}(\text{observation} - \text{estimation})^2} \tag{11}$$

MAPE is a measure of the estimation accuracy as a percentage (%). A small MAPE value is evidence for accurate queue estimation. RMSE is a measure of estimation stability in vehicle count. A small RMSE value indicates a high degree of estimation reliability.

The estimated results collected from the previously mentioned 10 replications are sorted according to the actual queue size (or number of vehicles in the link), from 0 to 18 vehicles. Figure 7 shows the MAPE and RMSE results with respect to the different numbers of vehicles in the off-ramp. When the queue size is relatively small, between one and nine vehicles, the MAPE is consistently high, whereas the RMSE remains low. The RMSE increases as the queue size increases. These results imply that the estimation accuracy and stability are compromised for short queues. As the queue size increases (larger than nine), the RMSE and MAPE continue to decrease, which indicates improved accuracy and stability of the queue estimation. In conclusion, the algorithm demonstrates stable and accurate queue estimation for intermediate and long queues between 11 and 18 vehicles on the off-ramp.

Sensitivity About Estimation Interval

The queue estimation interval affects the algorithm performance because the queue dynamics in the signalized link are determined mainly by the downstream signal control. Figure 8 displays the RMSE and MAPE with different estimation intervals. The presented results are collected from one simulation replication of the left-turning lane. The results illustrated in Figure 8 reveal that the estimation accuracy improved proportionally to increased estimation interval. With relatively shorter estimation intervals between 2 and 15 s, relatively larger errors occur because the estimations are dependent largely on the random gaps between vehicles.

However, with the longer estimation interval between 20 and 280 s, the estimation results demonstrate highly accurate and reliable estimations. The reason is that the aggregation of the occupancy measurements over time flattens the fluctuations caused by the traffic signal control and the random vehicle behaviors.

CONCLUSION AND DISCUSSION

This study investigated the relationships between locally measured time occupancy and indirectly measurable density. By taking advantage of the relationship as well as the distribution of time occupancy over signalized off-ramps, a queue estimation method has been developed.

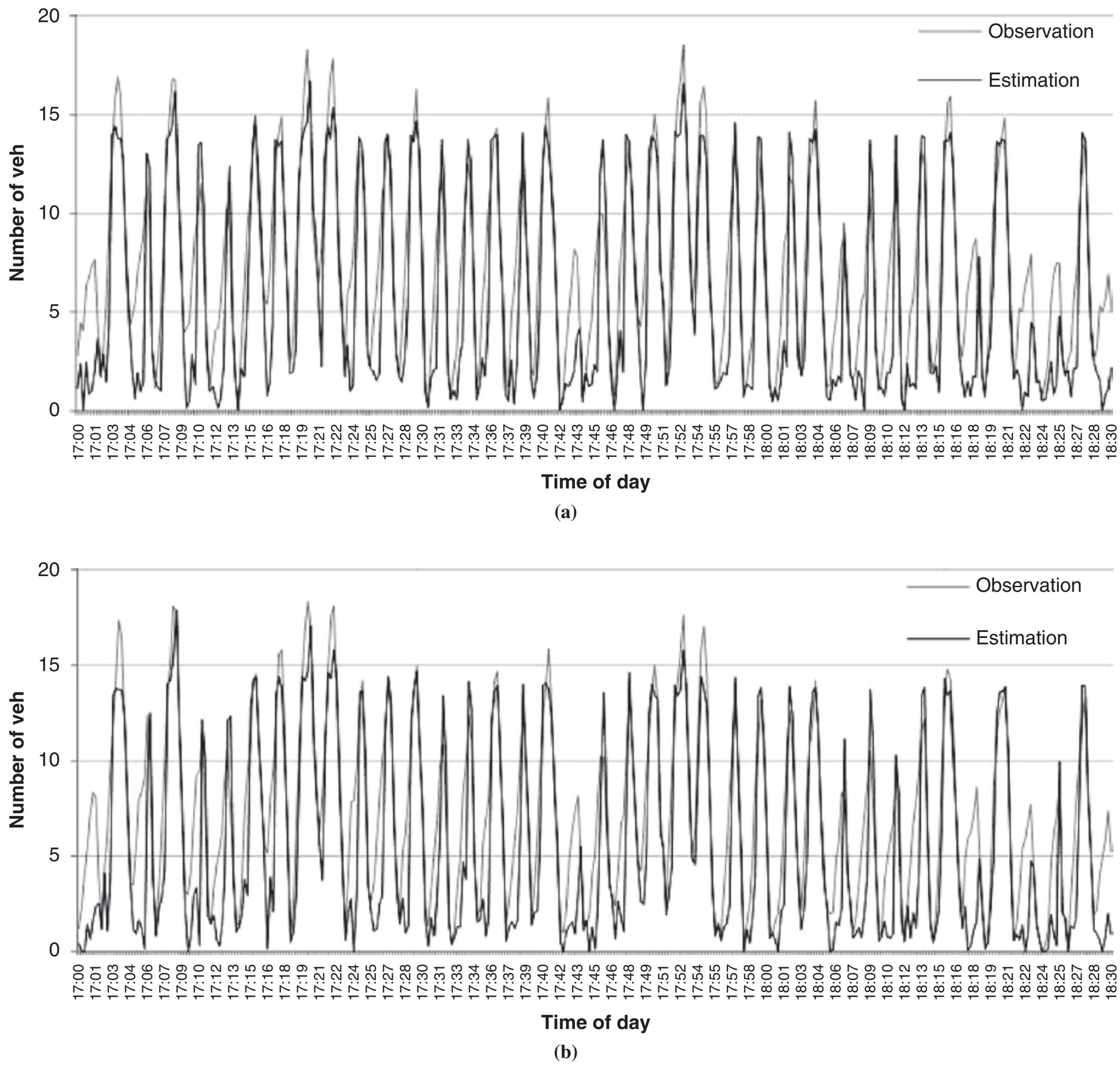

FIGURE 6 Comparison of actual versus estimated link densities: (*a*) Replication 3, off-ramp left lane, and (*b*) Replication 3, off-ramp right lane (veh = vehicles).

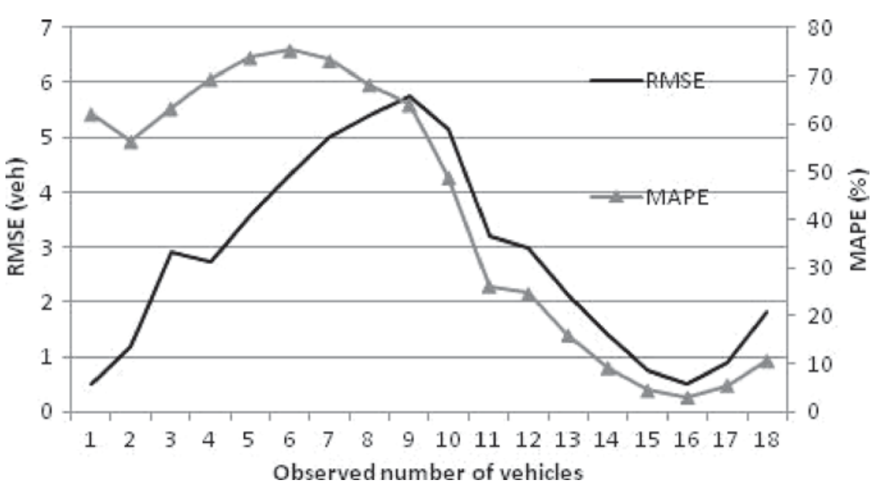

FIGURE 7 RMSE and MAPE for vehicle counts in link.

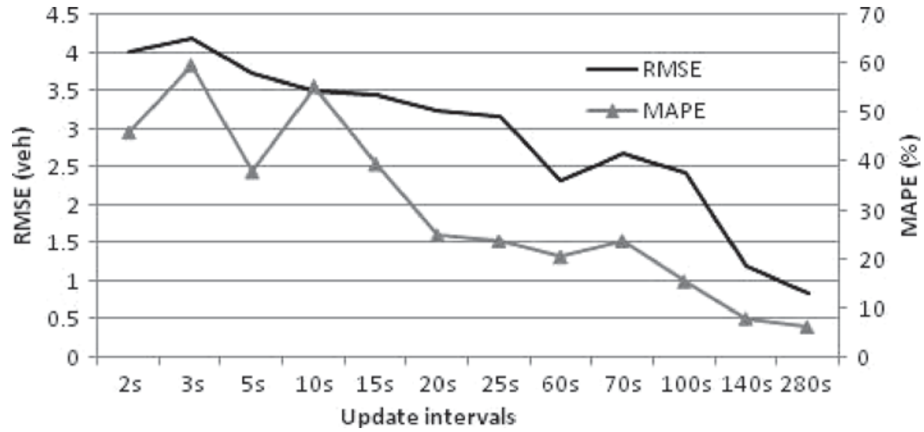

FIGURE 8 RMSE and MAPE for estimation intervals.

The prototype of the estimation algorithm demonstrated satisfactory estimation accuracy and reliability in the simulation tests. The estimation scheme performed well especially for long queue (congestion) scenarios. This method relies on time occupancy data, which are considered to be more reliable and bias-free measurements and thus make the algorithm more practical. As such, the estimation method can be applied to monitor traffic conditions for real-time traffic control systems. Future improvements to this method include expanding the scope of application to motorway on-ramps and urban signalized links. Some issues for implementation will be explored and further investigated in future studies.

The proposed method has shown relatively unsuccessful results in capturing short queues mainly because of the current detector setting, which is designed for long and intermediate queues. In cases in which short queues are a more important measure, the middle-block detector can be relocated downstream closer to the stop line. However, such a modification will generate a trade-off between the short queue estimation accuracy and the long queue estimation accuracy. The decision should be made depending on the queue size of interest.

The simulation tests revealed that the estimation interval directly affects the accuracy of the queue estimation results. Although relatively shorter estimation intervals still can be used for detection of queue occurrences and their developing patterns, longer intervals will be more useful in capturing the accurate queue status. The selection of estimation intervals must be determined on the basis of the information of interest or the control strategy objectives as well as data availability.

As discussed earlier, a large variance in vehicle lengths will create estimation errors because the linear coefficient α converting space occupancy to density is affected. In this situation, further traffic composition information in regard to vehicle lengths of different classes and the proportion of each vehicle type at the site will be required.

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